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Strategic Learning in Primary Mathematics Education: Effects of an Experimental Program in Modelling

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ABSTRACT

In most strategy research the focus is on ready-made models provided by the teacher or textbook. However, in this research project the effects are described of an experimental program in primary math education, concerning the construction and use of models by pupils in guided co-construction. This field experiment with an experimental group and a control group involved 238 grade-5 pupils. In a series of experimental lessons, pupils were taught to design models as a tool in the learning of percentages. The results of the experimental program were compared with the outcomes of a program in the control group, based on the teachers' strategy of "directly providing models" to the pupils. The conclusion, then, is that children in the experimental condition significantly outperform children in the control condition.

INTRODUCTION AND THEORETICAL BACKGROUND

Over the past 2 decades, there has been growing interest in the use of strategies and models in primary math education (Davydov, 1988; Gravemeijer, 1994, 1997). A major issue is the way in which pupils learn to work with models in the process of acquiring knowledge in a conscious and strategic way (Taconis, 1995). Traditionally, most research projects studied how pupils learn to work with ready-made models, provided by the teacher or the textbooks (Mayer,

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1989). But seen in the light of recent developments in realistic math education theories and socioconstructivist theories, pupils' contributions are increasingly considered highly important for the learning process. However, with regard to the actual educational use of strategies and models, this insight has not pervaded the mathematics classrooms yet. Therefore, this study will describe and analyse the effects of an experimental program in primary math education, concerning the use and construction of models.¹

Many studies have shown positive effects of the use of strategies in learning on academic achievement (Alexander, Graham, & Harris, 1998; Ertmer & Newby, 1996; Hoek, Van den Eeden, & Terwel, 1999; Weinstein, 1994; Weinstein, Husman, & Dierking, 2000; Weinstein & Mayer, 1986). In our research program, the "strategic learning" notion links up primarily with the work of the Dutch educational psychologist Van Parreren and as such it is also closely connected with the activity theoretical interpretation of human activity, development and learning (see Leont'ev, 1978; Van Oers, 1990; Van Parreren, 1993). We distinguish two types of actions within strategic learning:

1. Actions that are directly performed on material or mental objects, aiming at the change of those objects in the direction of a goal (performance actions);
2. Actions that deliberately regulate these performance actions in accordance with the actor's interests, motives, plans, goals (regulatory actions).

Firstly, strategic learning implies that pupils follow a certain procedure or method on the basis of an analysis of the problem situation. Here one can think of approaches, techniques, algorithms or heuristics to solve a problem. As part of such procedures, the heuristic representation of the problem situation by the choice or design of models is considered to be an important element. According to Van Parreren, strategic learning examples are "learning how one solves word problems, writes an essay, understands a difficult text or throws a ball as far as possible" (Van Parreren, 1993, pp. 58, 59, 64, & 71). Characteristically, in these examples there is no unique and fixed method to be acquired in order to reach the goal. The performance always requires exploration to some extent.

Secondly, the actions mentioned under point 2 are called strategic actions, because they coordinate, organize, and regulate the performative and

¹The study is part of the research program entitled "Strategic Learning in the Curriculum" initiated by the Department of Education and Curriculum at the Faculty of Psychology and Education of the Vrije Universiteit Amsterdam.

exploratory actions towards the goal. Hence, strategic actions can also be conceived of as a kind of *meta-actions*. Strategic learning, then, is defined as learning on the basis of such regulatory actions, or as learning these strategic actions themselves. To put it more concisely, one could say that strategic learning amounts to an activity-theoretical version of reflective learning or meta-cognitive learning.

Van Parreren states that both kinds of actions are not to be disconnected. It is precisely for this reason that in our research program strategic learning is conceived as embedded in the curriculum and as connected to domain-specific knowledge and concepts.

In this approach to learning, the knowledge acquisition process is seen as a constructive process, based on the production of new symbolic constructions that acquire actional meaning (Van Oers, 1996, 2000) for the deliberate regulation of goal-directed actions. The meaning of the new symbolic constructions is basically derived from a social process of meaning negotiation.

Sociocultural theorists argue that meaningful knowledge construction is only possible if pupils have the opportunity to work together in a cultural practice and learn from each other in that context. In this view, collaborative learning under teacher supervision is seen as a basic pattern for the organization of learning processes. The joint activity can be conceived as a kind of "distributed cognition" (Cole & Engeström, 1993), in which each participant can profit from cultural resources which are offered by other participants and by materials used in the activity. These resources enable each participant to accomplish more than they could do on their own. In this way, participating in such activity can be seen as jointly constructing a zone of proximal development (see Moll & Whitmore, 1993; Van Oers, 1995). In the higher grades of primary education the joint activity is increasingly becoming a discipline-based learning activity (Davydov, 1988), in which pupils are stimulated to develop and compare their own problem-solving methods. The negotiation of possible approaches and ways of problem solving will then contribute to the development of a solution supported by all participants (Cobb, Wood, & Yackel, 1993; Inagaki, Hatano, & Morita, 1998).

Aspects of the sociocultural perspective, as described above, can also be found in the instructional design theory of realistic mathematics education in The Netherlands (Gravemeijer & Terwel, 2000). Following Freudenthal (1991), we assume that guided reinvention of mathematics by pupils in a community of learners plays a major role. How does this active knowledge construction in a "community of learners" take place? Research into

mathematics education has led to the development of an approach in which the learning of mathematics is conceived as construction of meaning and understanding. This meaning formation is, however, impossible without the construction of new tools for communication about those meanings, that is, it needs the construction of new symbols (see Van Oers, 1996, 2000). In mathematics education, this often implies the construction of symbolic tools, based on the modelling of reality (De Corte, Greer, & Verschaffel, 1996). Together with the idea of math as both an active and a constructive process, the social character of this process is underlined. The pupils are not to be seen as isolated information processors (see Van Oers, 1990), but as participants in the class as a (mathematical) community (Cobb et al., 1993).

In realistic mathematics education, two strategies are used in order to challenge children to participate in this reconstruction process. First, bringing pupils into situations that make sense to them and provide them with opportunities to experience how mathematics was developed in cultural history. Second, encouraging pupils to produce spontaneous, informal, self-invented problem solutions (Gravemeijer, 1994). Pupils' informal strategies can often be interpreted as anticipating more formal procedures. Mathematizing such solution procedures creates opportunities for a reinvention process (Gravemeijer, 1994).

Nowadays, many researchers in the mathematics community are inspired by socioconstructivism (Cobb & Bowers, 1999; Cobb et al., 1993; Gravemeijer, 1997). Socioconstructivists argue that all knowledge is self-constructed in an interactive process with others. Hence, in mathematics lessons there must be a climate of open dispute, encouraging pupils to generate solutions and answers that make sense to them. In such a climate pupils must pool their self-invented solutions and interpretations in order to create a common sense that constitutes the basis for their community, for their intellectual exchanges, and for their negotiations of meaningful solution(s) to the problem(s) at hand (see Edwards & Mercer, 1987). In this process, it is the teacher's task to stimulate the pupils' acculturation process into the wider mathematical community and by doing so to support the pupils' understandings (Cobb et al., 1993).

Within the socioconstructivist approach, mathematical tasks are often embedded in day-to-day contexts. In schools, pupils do not always have access to skills needed to solve context-bound tasks. They experience problems with the application of newly learned actions to new situations, although they seem to master the basic actions. This problem of the strategy application in

mathematical problems in day-to-day contexts is frequently mentioned in the literature (Säljö & Greer, 1997). We interpret this phenomenon as a consequence of the lack of strategic learning abilities, particularly a lack of the construction and use of models for the regulation of problem-solving actions. Several strategies needed for the solving of context-bound tasks are discussed in the literature (Rosenshine, Meister, & Chapman, 1996). A possible solution to this problem might be a training program in social and cognitive strategies, in which designing and modelling would be crucial (Hattie, Biggs, & Purdie, 1996; Hoek, Terwel, & Van den Eeden, 1997; Hoek et al., 1999).

Modelling is seen as a powerful method for the structuring of open and ill-defined problems. Models can be seen as a tool, for example for building a bridge to overcome the gap from concrete situations to abstract mathematics. As such, a model is a tool for strategic thinking. It structures thinking: It is a symbol that refers to cultural historical knowledge and guides thinking to possibly successful solutions (Davydov, 1990). A symbol can be used for communication and discourse. However, a problem commonly found in the use of models that are developed by teachers or designers of methods is that pupils do not always grasp immediately what adults see in it. This is the case when models do not properly fit in with the pupils' own way of structuring a problem situation. In this case, a model should support the pupils' own exploratory actions, instead of transmitting the adult's way of performance.

In our view, modelling can be embodied in a variety of ways: It can take the form of making a drawing, a diagram, a formal-symbolic representation, or a story. In any case, it implies that an object (a situation, an action, a thing) is represented in a simplified way. The model representation articulates the structural aspects of the object involved, and obviously it holds the pretension of replacing that object for the time being. In self-invented models, the subject thus expresses his or her view on the object involved and as such also expresses his or her informal appraisal of the situation. This informal model is a stepping-stone for bridging the gap between informal (personal) representations and formal, mathematical representations.

It is assumed that models are useful for learning mathematics (Gravemeijer, 1994, 1997; Mayer, 1989, 1996, 1999). It is, however, doubtful whether the imposition of ready-made models on pupils' thinking, will help them master the mathematizing activity. Following Freudenthal (1991), we assume that the appropriation of the mathematical structuring ability might be more helpful than the mastery of mathematical structures. To put it differently: We advocate math as a human activity rather than math as a ready-made system. Therefore

we wondered if it might be more rewarding if we taught pupils to design models themselves, in co-construction with peers and the teacher (Gravemeijer & Terwel, 2000). The idea behind this point of view was that pupils who learn to design models in co-construction would choose to make informal models that are more in harmony with their competence level and their day-to-day understandings. Therefore, we suppose that it should help them to better understand the subject matter the model is about. Having learned how to construct adequate and more formal models presumably places the pupils in a better position for solving new problems for which no ready-made models are provided or available.

The optimal way for the appropriation of the modelling activity in the context of mathematical problem solving is still a matter of debate. In our research we want to shed some light on this process and its value for the development of mathematical thinking.

In the process of problem solving, pupils initially construct informal models, which are commented upon by their peers or teachers. As a result the pupils are stimulated to modify their models and improve them in order to bring them more in accordance with the consensual view or the requirements of the situation (Van Dijk, Van Oers, & Terwel, in press). By doing so, as Gravemeijer (1994, 1997) has pointed out, the pupils gradually reconstruct their model *of* a situation into a model *for* the situation. We theorized that by doing so pupils will gradually appropriate the mathematizing process (i.e., the activity of mathematical *structuring*) more profoundly than by adopting ready-made mathematical *structures* and performance actions.

RESEARCH QUESTIONS AND HYPOTHESIS

To test the above hypothesis, a study was conducted in which pupils were taught to work with models according to the principle of guided co-construction. This meant they learned to design models as a tool in the learning of mathematical actions. As mathematical subject matter we chose percentages and graphs. In our view, pupils should be seen as co-designers of models. Co-designing may lead to an upgrade in their level of thinking, caused by reflection and discussion during the co-construction of knowledge. The teacher essentially takes a crucial role in this process as one of the participants in the activity. The teacher guides the construction process by asking critical questions, making objections, providing additional information or even

suggesting hypothetical solutions when the pupils' solution process fails. In brief, the teacher guides the strategic (reflective) learning process and promotes the construction and reconstruction of models.

The study's research question is: What are the effects of acquiring models by co-constructive learning as compared to the mastery of models by an expository teaching approach? This co-constructive learning approach can be characterized as a form of activity in which the pupils participate as model designers in a mathematical context, and jointly construct mathematical models for the solution of complex problems.

Our *hypothesis* is that this teaching method will have positive effects, such as changes in learning processes and better learning results of pupils in mathematics. The rationale behind this hypothesis is that pupils will gain deeper insight into math problems structures because they will have gained experience in designing and will have been involved in strategic learning processes, in which exploration, reflection and negotiation play crucial roles. We expect this deepened insight to help pupils see through structures of new and unfamiliar problems. The strategy acquired can be used to solve new problems.

The main variables and their relationships in the study are represented in Figure 1. In this article we will focus in particular on the influence of the intervention and pupil characteristics on the learning outcomes.

Pupil characteristics are measured by a pretest. Learning outcomes are determined by a posttest that measures the knowledge pupils have acquired afterwards. The *Teaching and learning processes* were described in protocols on the basis of direct observations and video recordings. *Program* refers to the

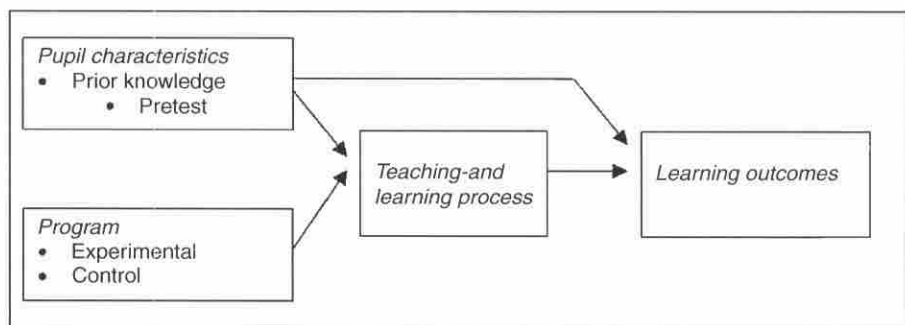


Fig. 1. Main variables and their relationships.

intervention, which will be described in more detail in the curriculum materials section. The program consists of 13 lessons.

METHODS

Research Design and Participants

In this field experiment, with an experimental group and a control group, 8 schools, 10 classes, 10 teachers and 238 grade-5 pupils (age 10–11 years) were involved. A pretest-posttest control group design was used. 117 pupils were assigned to the “providing” condition: Teacher-made models were provided to the pupils while they were learning the percentage-concept. This “model providing” approach is the way in which regular education takes place in most primary schools in The Netherlands. The experimental (“co-constructing”) group, consisting of 121 pupils, was exposed to the same mathematical content, but here the emphasis was on “guided co-construction” of models by pupils and teacher.

The schools participating in the experiment were situated in, or near, two cities in the centre of The Netherlands. Experimental and control schools were either middle-class schools or schools with high proportions of ethnic minority group children. Both types of schools were equally represented in the two conditions.

The grade-5 classes were randomly assigned to the control condition or the experimental condition. At the schools that participated with two teachers and two classes, one class was assigned to the control condition, and the other class was assigned to the experimental condition. There were no “drop-outs” (teachers or classes) during the study.

Curriculum Materials

For the purpose of this study a series of lessons was developed on the subject of percentages and graphs. Given our theoretical assumptions, this purpose, however, cannot be achieved with any curriculum. As Gravemeijer stated:

A teaching strategy that leads to comparing and explaining solutions by pupils is only possible if the learning sequence consists of contextual problems that give rise to a variety of solution procedures. It is the variety that allows for discussions about adequacy and efficiency, which in turn leads to a reflection on these procedures from a mathematical point of view. (1994, p. 90)

The tasks and assignments to be done in the lessons were open, complex problems. They were chosen from current math methods used in The Netherlands (Pluspunt, Wereld in Getallen Nieuw) and in the United States (Mathematics in Context). Furthermore, exercises were used which had been developed for MILE (Multimedia Interactive Learning Environment), a project of the Freudenthal Institute for teacher trainees. All the exercises used were adapted to fit the purposes of the intervention intended. In addition, a teachers' manual was developed containing the main ideas and principles of the approaches.

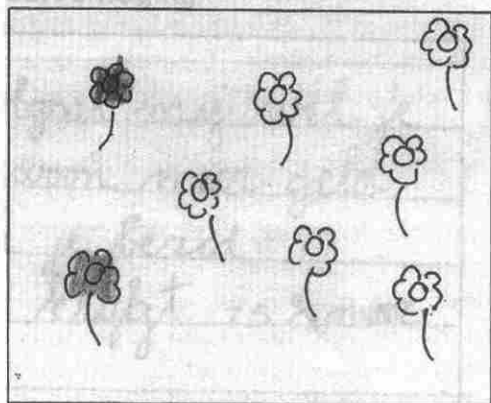
The intervention consisted of a 1-hr lesson every day for (almost) 3 weeks. It consisted of 13 lessons: one lesson in which the children learned about strategy use, models and their functions, and 12 lessons on percentages and graphs. Four lessons dealt with percentages, another 4 lessons dealt with graphs. In the rest of the lessons, percentages and graphs were combined. To improve the readability and save space in this article, we have chosen to show examples of the percentages lessons only, although tasks of graphs have taken an equal amount of time in the lessons and the tests. Of course, the posttest also contained both topics.

In both conditions, the same amount of time was spent on the tasks. The teachers limited their lessons to 1 hr a day, not only to keep the conditions alike, but also to keep the attention and concentration of the children as high as possible. The mathematical content in both conditions was the same, but the method (providing or designing) differed, as will be elaborated in the curriculum materials paragraph.

For each condition a particular version of the program was made. These versions show a number of essential differences. Firstly, in the way pupils learned to work with models: In the experimental condition the pupils co-constructively were encouraged to design their own models, as a tool for the learning of percentages and graphs. In the control condition the pupils learned to apply ready-made models provided by the teacher. They did not learn to design or choose models themselves. Secondly, the children in the experimental condition were asked to work in pairs and to discuss the models they had designed, before the models were discussed in class. The children in the control condition worked individually on the tasks.

The following is a typical example of the kind of task that was assigned in both conditions. In some assignments pupils were asked to recognize percentages and shade this amount on a given model (provided condition) or invent a representation (designing condition) of this amount. An example can be found in Figure 2.

25 % van de bloemen zijn rood



25 % van de bloemen zijn rood

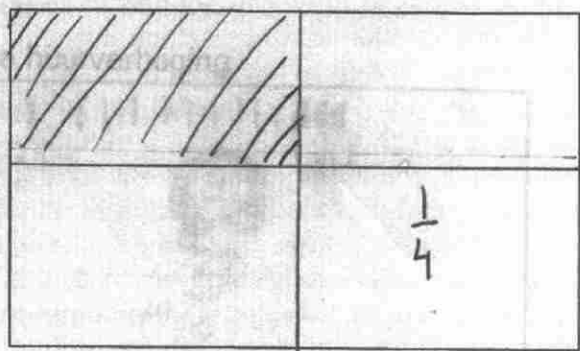



Fig. 2. Examples of a task in the two conditions. First the results of the task in the providing condition in which the model (here: 8 flowers) is readymade and second a task in the designing condition: the model was invented by one of the pupils. The text above the assignment reads: "25% of the flowers are red."

Other tasks, such as the coffeepot assignments, were more open-ended and ill structured (see Fig. 3). In this task the teacher told a story about a luxury cinema, where during the movie one could ask for coffee by pulling a button.

5  *Is er genoeg koffie?*

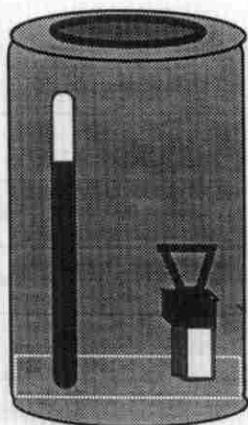
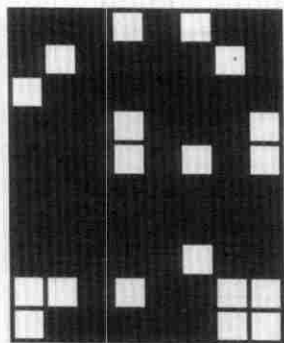
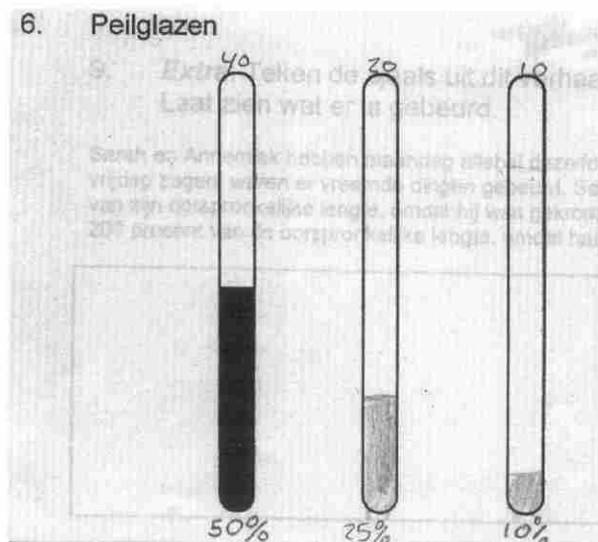


Fig. 3. The introduction of the coffee pot assignment in both conditions. The text above the task means: “Will there be enough coffee?”

The images in Figure 3 show a seating plan in the cinema and a coffee pot that is filled for three quarters.

The question is whether there is enough left to provide everyone who pressed the button with one cup of coffee. The pupils have several decisions to make: What do the black and white blocks mean on the seating plan? How many cups of coffee does the pot contain? Most pupils decided that the black blocks stood for people who wanted coffee and that a whole coffee pot should contain enough cups to serve everyone in the cinema once: that would be 80 cups, as can be seen on the seating plan. About three quarters of the pot is filled, so about 60 cups of coffee are available. This is not enough coffee for the 61 coffee-drinking people. In this task, the relation between fractions and percentages was explored and explained when necessary. See Figure 4 for examples of the models the children designed, or were provided with, in the cinema context.

Indeed, one could argue that it is possible to solve these problems with already existing knowledge of fractions only, and without any use of model construction. Our point of view is that percentages and fractions are conceptually interwoven, and that it is very difficult (if not impossible) to learn



6. Peilglazen
- Kun je in een zelfbedachte tekening aangeven hoeveel koffie er nog in de kan zit?
- A. Nog 50% C. nog 10%
- B. Nog 25% D. Extra: nog 80%

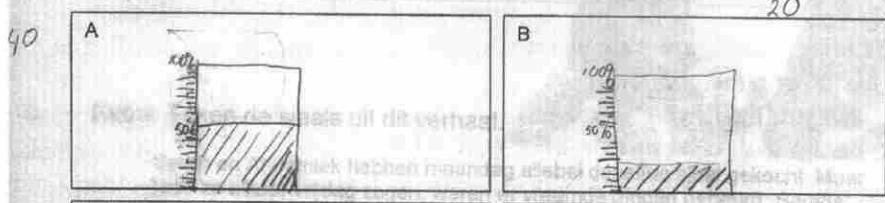


Fig. 4. Differences in model use. First an example of a task in the providing condition in which the model is provided and second a task in the designing condition in which the pupils invent representations themselves. The text above the assignment reads: "can you make a drawing, which shows how much coffee is left in the pot? A. 50% coffee left B. 25% coffee left."

about percentages without some earlier knowledge of fractions. Therefore, in the teacher's manual we emphasized the need to explicitly discuss the connections between fractions and percentages. We expect pupils in the experimental condition to be better able to master the conceptual basis of percentages and graphs, due to their work at modelling. We suppose that their

learning process of “how to model a situation” leads to a deeper insight, and more flexibility. As a consequence, this leads to better results on the posttest for the experimental group, even when they only use their knowledge of fractions. Considering the difficulty level of the problems in the test, we expect that the solution of these problems would have been impossible, merely on the basis of rote knowledge of fractions. Therefore, we assume that a higher score of the experimental group is due to an improvement of insight into model making, and consequently of a better use of fractions and percentages.

Classroom Activities

We will now further elaborate on the kinds of classroom activities that occurred in both conditions. We will start with a description of the classroom activities in the providing condition, afterwards we will continue with the designing condition. The coffeepot assignment, already discussed in the last paragraph, will be taken as an example.

After the introduction of this assignment the pupils of the providing condition worked with a ready-made model of the pot with various coffee amounts left. They were asked to explain in percentages (50, 25, 10, and 80%) how much coffee remained and how many cups of coffee were left in the pot. The teacher walked around and gave help to pupils who needed it. Their model use and solutions were then discussed in a class discussion. A protocol is given of the last exercise (80% of the coffeepot is filled), which shows how teacher and pupils worked on the assignment.

Teacher: The last task was the most difficult one. Danny, what do you think? How many cups of coffee does the pot contain when it is filled for 80%?

Danny: One cup ..

Kids: Huh?

Teacher: One cup? So if you serve one cup of coffee, your pot will be empty? (Pointing to the model with 80% on it) But we saw in the first model that when the coffeepot contains 50% coffee, we have coffee left for 40 people. Let's help Danny. Fenna, what do you think?

Fenna: 64 cups.

Teacher: 64. Fenna, we would really like to know how you found this amount.

- Fenna: Well, if you know that 50% is half, you just do plus 8, plus 8 and plus 8.
- Kids: How? I don't understand! I don't get it!
- Teacher: Wait, I do understand what she's saying! Look at this (points at an earlier model): 50% was the same as 40 cups. And we also calculated what was 10%, that was 8 cups. So Fenna did: 50%, plus 3 times 10%, which is 80%. So she did: 40 cups, plus 3 times 8 cups, is 64 cups of coffee. Any one who did it another way?
- Ben: I did 8 cups, which is 10%, 8 times.
- Teacher: Very well!

In contrast, the pupils in the designing condition were asked to develop a drawing in which one could see that the pot contained 50% (or 25, 10, and 80%) of the total coffee amount. The teachers' way of working let the pupils themselves think about the problem solution first, without giving them ready-made solutions. They were stimulated to visualize and represent the situation in a model. No ready-made model was provided and it should be noted that teachers in the designing condition did not ask the pupils to use a certain model. While the pupils were designing their models, the teacher walked around the classroom. The teacher helped where necessary, and asked critical questions to individual children about the applicability of the designed models. The pupils were stimulated to discuss their models with their neighbours, and to make improvements. During this process, the teacher asked several pupils to draw their model on the blackboard. When most pupils had at least thought of a way to represent the problem, the teacher started a class discussion. The researchers expected that children who saw the advantages of one of the models discussed, would eventually adopt it on their own initiative. A protocol of the designing condition is given below. It starts at the point where several models on the blackboard are discussed. Each of these models was meant to represent the coffeepot, which was for 25% filled with coffee.

- Teacher: Bart, first we'll take a look at your model. Can you explain what you did?
- Bart: Yes, I drew a coffeepot. (It's a coffeepot in a concrete way, complete with handle and other accessories.)
- Teacher: OK, and how can we see it is for 25% filled with coffee?

- Bart: (Points to his model) This little line, at half of the pot.
- Teacher: Aha, but you also made a line at 50% and 75%. How can I see that you mean it is for 25% filled?
- Bart: Oh, I forgot! (Quickly shades at the blackboard the lowest quarter of the coffeepot.)
- Teacher: That's better. Ann, tell us about your ideas?
- Ann: I made a sort of thermometer, and I put 100 small lines in it. And then I coloured the first 25 lines red.
- Teacher: OK, and how did you figure out how many cups of coffee this pot contains?
- Ann: I knew that 25 out of 100 is a quarter.
- Teacher: Can you show it on your model?
- Ann: (Points at her model).
- Teacher: And how did that help you?
- Ann: Well, I knew the total pot contained 80 cups, and then I took a quarter out of 80.
- Teacher: Good. Did it take you long to draw this model?
- Ann: Uh... yes, quite long...
- Teacher: Who can think of a solution to make this model easier to draw?
- Jesse: I would not draw all those 100 lines, but only the most important ones, like 0%, and 100%.
- Teacher: OK, that would make it much easier. Linda, can you explain your model?
- Linda: I just made a kind of a bar. That's supposed to be the coffeepot. And then I shaded the first quarter of it.
- Teacher: Why did you choose a bar to represent the coffeepot? It doesn't look like a coffeepot at all!
- Linda: Because the model doesn't really have to look like a coffeepot. I think a bar like this is just easy to draw! And you can use it in every situation.
- Teacher: Well, we saw three models. Who made a model like Bart's? (Several children raise their hand) And who has made a model more like Linda's? (A couple of children raise their hand). Jim, you didn't raise your hand. What model do you like best, Bart's, Ann's or Linda's?
- Jim: Linda's...
- Teacher: Why?

- Jim: Because it's fast, you don't need to draw the whole coffeepot. And you don't need to draw 100 lines, like in Ann's model. Linda's is simpler.
- Teacher: These models on the blackboard are all OK, but they do differ at some points. Some models are easy to use in this situation, but also in other situations. And we agreed earlier that a model has to be as simple as possible, without all kinds of extras. And that it is important that you can draw it quickly. That's why Linda's model is a good one. You can use it also in other situations, even when it is not about coffeepots, it is simple and it is quick.

In the providing group, as could be seen in the first protocol, the teacher stimulated the children to refer to the model provided while explaining their thinking. They gave their explanation in words and the teacher pointed at the model on the blackboard. They were expected to apply the model provided in their solutions of the exercise.

As the protocol of the designing group shows, some of the pupils in the designing condition were given a moment to explain their model and their thinking to the other children. The others could suggest improvements, and the teacher asked critical questions to stimulate reflection and to provoke improvements. The whole group learned about the process of model designing, and how to use models to solve math problems. In short, the difference in working with models in both conditions is that in the providing condition children are asked to use models that already exist without changing these models to their own needs, while in the designing condition the children use models they reinvent themselves and therefore can change models they like for use in other situations.

Procedures

The experiment started with teacher interviews, by which the researcher could gain insight into aspects such as teacher attitudes and perspectives on mathematics, collaborative learning and other important characteristics of the individual teachers. Then, for each of the conditions separately, a workshop was organized in which the program and the teacher manual were explained and materials were discussed. The teachers participating in the experiment joined one of the two workshops. In the autumn of 1999, all teachers started the program in the same week and ended the program 3 weeks later.

All teachers were visited at least twice in the course of the investigation, to give them opportunities to ask questions about the material and to give the researcher the opportunity to control the course of the lessons, the validity of the conditions and the integrity of the intervention. In addition, the teachers filled out a small questionnaire after each lesson. At the end of the program a second interview was held. This interview would give the researcher an impression of the teachers' evaluation of the program implemented.

Instruments and Analyses

A broad spectrum of methods and instruments was used to describe and measure the teaching-learning processes and outcomes. We made direct, participant observations, made audio and video registrations, administered teacher and pupil questionnaires, and pre- and posttests (standardized as well as curriculum-specific tests). In this article the analysis is restricted to pre- and posttests. Just as examples, a few observation samples of the teaching-learning processes were added.

We administered a standardized pretest and a curriculum-specific posttest, in order to find effects of the program in terms of learning results. The tests proved to be reliable with alphas ranging respectively .83 and .90. It should be noted that the alpha of the standardized pretest is based on oral reports of the institute that developed the test and that no publications are available yet.

The posttest measured the pupils' achievement regarding percentages and graphs in a quantitative way. The answers were assessed as either "correct" or "false."

RESULTS

The data presented in this article were analysed with the statistical program SPSS, and MLwiN. In order to determine the intervention effects, one-way ANOVA, regression analysis and effect sizes were used. In Table 1 the characteristics of the pre- and posttest distribution are presented.

From Table 1 it can be seen that the experimental group gains more on the posttest than the control group. In a one-way ANOVA no significant differences on the pretest scores between the two groups were found (pretest: $F(1, 237) = .484, p = .487$). The difference in posttest score is 5.85 in favour of the experimental group. A one-way ANOVA resulted in significant differences between the posttest scores ($F(1, 243) = 8.203, p = .005$). The

Table 1. Characteristics of the Distributions of the Standardized Pretest and the Curriculum Specific Posttest for All Pupils (N -Pupils = 238, N -classes = 10).

	<i>M</i>	<i>SD</i>	Min	Max
Control program N -pupils = 117				
N -classes = 5				
Pretest	136.07	22.02	49	189
Posttest	43.55	14.76	10	73
Experimental program N -pupils = 121				
N -classes = 5				
Pretest	137.87	17.80	95	181
Posttest	49.40	13.76	17	75

ANOVA with the pretest as covariate also shows significant differences on the posttest ($F(2, 235) = 208, 256, p = .000$). Hence, it can be concluded that in general there is a positive effect of the experimental program on learning results. This result is in line with the hypothesis of the study. No interactions were found, therefore it was allowed to proceed with regression analysis.

We considered the effects of the variable "intervention" and "pretest" on the learning outcomes. A multiple linear regression analysis was conducted, in which a dummy variable was created for intervention (0 stands for the control group [providing condition] and 1 for the experimental group [designing condition]). The variables pretest and intervention were added with the stepwise method. The outcomes are presented in Table 2.

Table 2. Summary of Multiple Linear Regression Analysis for Variables Predicting the Scores on Posttest ($N = 238$).

Variable	<i>B</i>	<i>SE B</i>	β
Step 1			
Pretest	.569	.030	.781*
Step 2			
Pretest	.563	.029	.774*
Intervention	4.927	1.137	.170*

Note. $R^2 = .61$ for Step 1; $R^2 = .639$ for Step 2;
 R^2 change = .029 for Step 2; ($ps < .05$).

* $p < .000$.

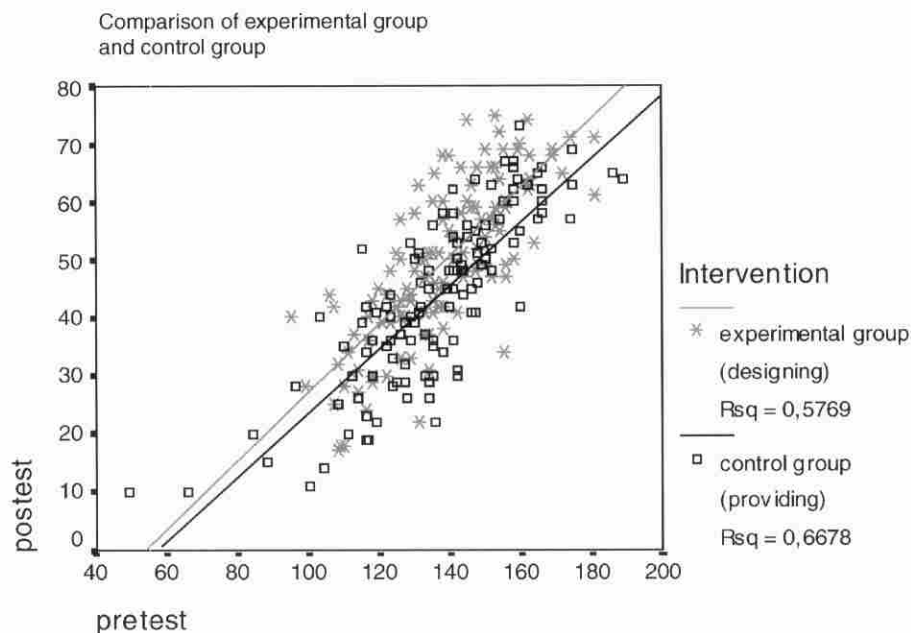


Fig. 5. Comparison of experimental group and control group.

The data distributions were checked for the assumptions of linearity, heteroscedasticity, and outliers. From Table 2 it can be concluded that the variable pretest already explains 61% of the posttest variance. The variable intervention contributes with another 3%. We decided to compare the regression equations of the two groups in this study. In Figure 5 we plotted the individual results of the children in the providing group and the experimental group on the pre- and posttest. For these tests we calculated the regression lines and the prediction intervals for single observations (confidence level = 95%). The regression equation of the control group is: $\text{posttest} = -31.027 + .547 \text{ pretest}$. The regression equation of the experimental group is: $\text{posttest} = -31.562 + .587 \text{ pretest}$. This means that all children profit from being in the experimental group. Although children from the control group also show gains, the “experimental group” slope is slightly steeper and the intercept is higher, as can be seen in Figure 5.

Effect sizes were calculated: this is the difference between the posttest means of the intervention group and those of the control group, divided by the standard deviation of the control group. The effect size in this study is .40.

This is considered as a relatively small effect (Cohen, 1988). Effect sizes of $+.20-.25$ are of practical significance in educational environments (Slavin, 1996, p. 31). An effect size of $.40$ means that if an average pupil (50th percentile) of the control group had been in the experimental group, this pupil now would have scored on the 66th percentile of the control group (from $.50$ to $.66$), which is a relevant gain (for an explanation of this computation, see Rosenshine et al., 1996).

In addition to the standard covariance analysis, which showed no interaction effects, a multilevel analysis was done, since it explicitly views that interaction as an effect of Condition at class level on the influence of the Pretest on the Posttest. The differences between the conditions could be due to a few classes in the experimental or control group. Although the number of classes is limited for this type of analysis, according to Snijders and Bosker (1999) this analysis is justified by $N = 10$ at class level and $n = 100$ at pupil level. Our sample size is $N = 10$ at class level and $n = 238$ at pupil level.

Before presenting the outcomes of the multilevel analysis some preliminary remarks have to be made about the meaning of the intercept and the slope coefficients that are shown in Table 3. In model 1, the variance in the dependent variable (Posttest) is divided in a component at pupil level (173.70 or 84%) and a part at class level (33.48 or 16%).

Table 3. Results of the Multilevel Analysis, Regarding the Posttest in Mathematics as the Dependent Variable.

	Model 1	Model 2
Fixed part		
<i>Pupil level</i>		
Pretest		.54 (0.03)
<i>Class level</i>		
Mean Pretest		-. (-.)
Condition		4.66 (1.69)
Random part		
<i>Pupil level</i>		
Variance	173.70 (16.31)	69.89 (6.56)
<i>Class level</i>		
Variance	33.48 (18.21)	4.04 (3.15)
$-2 \cdot \log(lh)$	1911.97	1687.67

Note. $p = .05$, Standard error between parenthesis, N -Pupils = 238, N -classes = 10.

'-' = Non-significant effects.

In model 2, three variables are introduced: Pretest at pupil level, and Mean Pretest and Condition both at class level. The coefficients in the fixed part can be viewed as the conventional unstandardized regression coefficients. For example, in Table 3, the coefficient .54 means that a change of one unit on the Pretest (preknowledge in Mathematics) scale will result in a change of .54 unit on the Posttest (Achievement in Mathematics) scale. The descriptives from Table 1 (means) can be used to estimate the relative magnitude of the effects for an average pupil by multiplying the coefficient by the corresponding mean for the Pretest.

Let us also give an example of a class level variable from Table 3 by referring to the coefficient 4.66. Pupils in the experimental condition get a learning gain ("bonus") of 4.66 scale points on their Posttest score.

The coefficients in the random part of Table 3 refer to the variances of disturbance terms that are left after introduction of the pupil and class variables in the analysis. To put it differently, the random part concerns the residual (unexplained) variance after having introduced all independent variables in the analysis. The variance left at pupil level is called within-class residual variance; the variance left at class level is called between-class residual variance. The outcomes of the multilevel analysis are presented in Table 3.

In the multilevel analysis it turned out that the slopes between the classes did not differ significantly. Hence, we decided to look further for differences in intercepts between the classes to be explained by differences in class mean on the Pretest (Mean Pretest). The result of this analysis also shows no effects. The final conclusion is that the outcome of the multilevel analysis clearly confirms the outcome of the earlier analyses at the individual level (regression analysis and ANCOVA). No interaction effects and no class effects could be found. The introduced variables in model 2 explained a large part of the variance on both levels. The non-significant contribution of the class variable Mean Pretest was unexpected. In several studies it was found that the mean class pretest score contributes to the scores at the Posttest (see for example Van den Eeden & Terwel, 1994). However, these studies concern secondary education, which means streamed classes. In primary education classes are not streamed, although differences between schools can be large. Therefore, further research with larger samples is needed. The results of Table 3 are presented in a graph in Figure 6.

We hypothesized that the pupils' learning processes in the experimental group would progress in such a way that a greater increase in results on the posttest would be demonstrated, as compared to the growth of control group

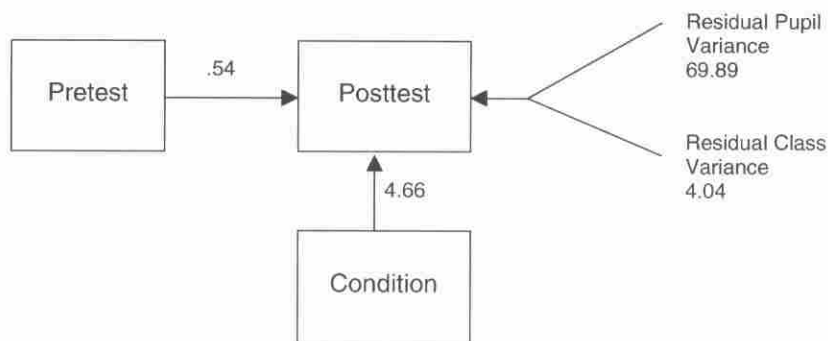


Fig. 6. Diagram of the multilevel Analysis.

pupils. The results confirm our hypothesis: Pupils in the experimental group significantly outperformed pupils in the control group on the posttest. Taking into account the ANOVA, regression analysis and multilevel analysis, we conclude that the experimental intervention had a significant, positive effect on the learning outcomes. Experimental pupils clearly show more progress in the learning of mathematics as compared to pupils in the control condition.

CONCLUSION AND DISCUSSION

This study focused on the effects of an experimental method in which children learned to design models and to draw graphs in co-construction, as a tool in working with percentages. In this article we addressed the research question: What are the effects of acquiring models by co-constructive learning as compared to the mastery of models by an expository teaching approach? We compared learning outcomes of pupils who had learned to work with percentages according to the method of “designing models,” to learning outcomes of pupils who had learned to work with percentages in a more regular way, with the teacher providing ready-made models for them.

In the course of this article we hypothesized that strategic learning, of which learning “how to model mathematical problems” is an example, would be beneficial for the solution of mathematical problems. We expected this strategy to have positive effects, such as changes in the pupils’ classroom activities, changes in learning processes, and better results of pupils on cognitive tasks. Pupils were expected to gain deeper insight into the math

problem structure because of their experience in designing and their involvement in strategic learning processes.

Elsewhere (Van Dijk et al., in press), we already reported an in-depth observation study on two teaching conditions on the pupils' classroom activities. In the present article we focused primarily on the effects of the conditions on the learning outcomes. The results clearly show the positive effects of the experimental program. In addition, we also had clear indications that these outcomes occurred on the basis of differences in classroom activities between the two conditions. Pupils who learned to construct models in the experimental program scored significantly better on the posttest than children who learned to work with models provided by the teacher.

How can these effects be explained? Although the outcomes of the study are in line with the expectations as elaborated in the theoretical background and the hypothesis, how can we substantiate that the intended teaching-learning activities really occurred in the classroom? Do we have empirical data about the implementation of the program, in this case the classroom activities? One limitation of this study is that the available resources for this project did not allow for systematic observation in all classes and all lessons. Causal inferences in the strict sense cannot be made because of the fact that we do not have quantitative observation measures for each of the 10 classes to introduce in the equation for the regression analyses, for example. Nevertheless, we have clear indications that the intended curriculum was realized in daily classroom practices and that the expected outcomes occurred as a consequence of these classroom activities.

In trying to make the intervention a success, we put a lot of effort in the preparation and planning. In preparation of the present study, a small-scale, qualitative case study was carried out. This case study – in which a “providing” and a “designing” classroom were contrasted, and in which two pupils were followed closely in their learning process – clearly showed the intended activities of teachers and pupils. In addition, some indications were found that the expected results of learning occurred. These in-depth descriptions and analyses of modelling processes were already reported elsewhere (Van Dijk et al., 2003). From this qualitative study we received a firm foundation for the present study and a convincing case that the intended processes can be realized in normal classroom settings.

As part of the present study, which is the main study of the project, curriculum materials and a teacher manual were developed, tested and revised. Teachers received a training and in turn teachers prepared and guided their

pupils in the co-construction processes of problem solving and designing models. During the experiment the researcher consulted each teacher in order to get insight into what pupils and teachers were doing in the classroom. All teachers were interviewed before and after the experiment. In addition, qualitative observations were carried out and video recordings were made.

In the "classroom activities" section of the present article, we included a few protocol samples of these observations and video records. These fragments clearly show the differences between the two conditions, in terms of the role of the teacher and the role of the pupils. The teachers in the providing condition guided the way in which ready-made models were applied, while the teachers in the designing condition scaffold the process of model designing. At this moment the whole qualitative data set is still in the process of being analysed and cannot be incorporated in full within the scope and limits of this article.

Although we have to admit that no decisive, quantitative proof can be given, we believe that there is convincing evidence from qualitative data that the intervention really occurred in the classrooms. The outcomes of the case study, the side measures which were taken to reassure an implementation according to the intentions, the classroom consultations, the interviews with the teachers, along with the samples (protocols) of the teaching and learning processes may substantiate our claim. Moreover, it is noteworthy to remind that our hypothesis was based on a firm body of theoretical knowledge and that there are some empirical findings from literature, which point in the same direction.

All in all, looking at the outcomes of the study and taking into consideration the strengths and limitations of our design and measurements, we tenaciously conclude that our hypothesis is confirmed. Pupils in the experimental condition outperformed their counterparts in the control group. Learning how to design models in the context of a strategy for solving mathematical problems in daily life situations, put pupils in a better position as compared to pupils who were provided with ready-made models. Our theoretical point of departure and the results of our experiment make us believe that designing models in co-construction may lead to a deeper insight into the meaning and use of models and consequently make possible a more flexible approach in problem solving.

Implications for Education and Future Research

This study has provided useful insights into the effects of the experimental program on pupils' learning outcomes. The empirical findings go in the direction of the theory presented and as such are a confirmation of the hypothesis. Although the results are significant, the program did not have a strong influence

on the pupils' mathematical achievement, as was shown in the regression analysis (3% of the variance was explained by the program, over and above the variance already explained by the pretest. The effect size was .40).

In this research, our resources were not sufficient to allow for more classes to participate in the experiment. Generalizing these results will be difficult, because of the limited number of classes ($N = 10$). It would be useful to repeat this study with a larger number of classes.

This article did not address the interesting question from a longitudinal perspective, of how pupils' models developed in the experimental condition during the program. In future articles the way in which pupils make the step forward from a "model-of" to a "model-for" approach (Gravemeijer, 1997) will be examined in a more qualitative way.

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